ADVANCED FUNCTIONAL MATERIALS

Supporting Information

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Multiplexed Nondiffracting Nonlinear Metasurfaces

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1. Theoretical Full Width at Half Maximum of Bessel beams

The scalar form of Bessel beams propagating along the z axis can be described in cylindrical coordinates (r,ϕ,z) by:

$$E(r,\phi,z)=A\cdot\exp(ik_z z)\cdot J_n(k_r r)\cdot\exp(\pm in\phi), \tag{S1}$$

where A is the amplitude, k_z and k_r are the longitudinal and transverse wavevectors. J_n is the Bessel function of the first kind and the term $\exp(\pm in\phi)$ represents the phase of orbital angular momentum. The full width at half maximum (FWHM) of the zeroth-order Bessel beam J_0 can be derived from Equation (S1) as:

$$\mathrm{FWHM}_{J_0}=\frac{2.25}{k_r}=\frac{0.358\lambda}{\mathrm{NA}}. \tag{S2}$$

Herein, the FWHM of the higher-order Bessel beam J_n is defined as the radius of annular ring, and given by:

$$\mathrm{FWHM}_{J_1}=\frac{1.84}{k_r}=\frac{0.293\lambda}{\mathrm{NA}}. \tag{S3}$$

$$\mathrm{FWHM}_{J_2}=\frac{3.06}{k_r}=\frac{0.487\lambda}{\mathrm{NA}}. \tag{S4}$$

$$\mathrm{FWHM}_{J_3}=\frac{4.20}{k_r}=\frac{0.669\lambda}{\mathrm{NA}}. \tag{S5}$$

2. Characteristics of the multiplexed Bessel beams for the depth of focus

The hyperbolic phase distribution of conventional focusing can be described as $\varphi_0(r)=k_0\sqrt{r^2+f_0^2}$, where k_0 is the linear wave vector, r is the distance of each meta-atom from the coordinate origin, and f_0 is the linear focal length. The first order approximation of Taylor expansion of this hyperbolic phase distribution can be express as $\varphi_0(r)=\dfrac{k_0 r^2}{2f_0}$, and the nonlinear focal length is given by

$f_n = \frac{2n}{n \pm 1} f_0$. Therefore, the transmitted SHG of spin σ and $-\sigma$ have nonlinear focal lengths of $4f_0$ and $4f_0/3$, respectively. The focusing characteristics of the conventional focusing is fixed due to the approximate linear relation between f and φ.

However, due to the depth of focus (DOF) and φ of a Bessel beam is nonlinear related, the DOF characteristics are more complicated as shown in Figure S1. The radius of the metasurface is 80 μm. This nonlinear relationship originated from the existing of the arc tangent function. Compared to the conventional focusing, multiplexed Bessel beams have a higher degree of freedom in controlling the linear and nonlinear focal lengths.

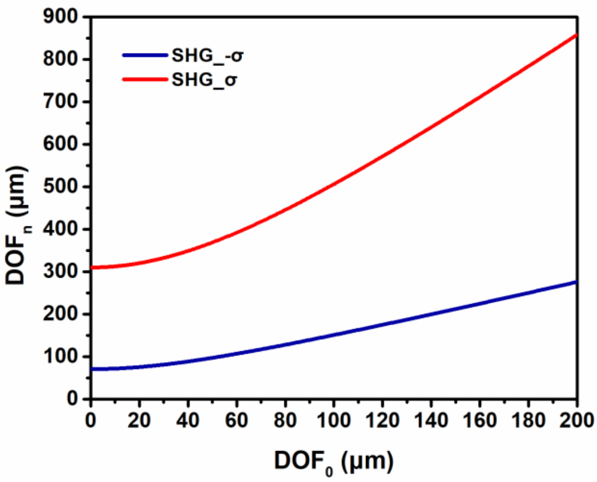

Figure S1. DOF characteristics of the multiplexed Bessel beams.

3. Correlation coefficient between the Airy function and metasurface

To confirm the feasibility of the approximation $\mathrm{Ai}(x) \approx x^{-\frac{1}{4}} \exp\left(iCx^{\frac{3}{2}}\right)$, we calculated the correlation coefficient between the wave function of metasurface $U(x) = \exp\left(i\left(-\frac{4}{3}a^{\frac{1}{2}}kx^{\frac{3}{2}}\right)\right)$ and the Airy function $\mathrm{Ai}\left(\left(2a^{\frac{1}{2}}k\right)^{\frac{2}{3}} x\right)$ with the same acceleration coefficient a = 2.5 nm^{-1}. The complex amplitude profiles of these two functions are presented in Figure S2a, although the amplitude information is lost, the oscillatory feature of the Airy function is completely preserved. Figure S2b shows the intensity cross profiles of the two functions in *k*-space, the scalar form of the Airy function leads to the symmetrical form in *k*-space. The computed correlation coefficient between them in *k*-space (from 0 to k_0) reaches a strong correlative value of 0.90 which confirms the feasibility of this approximation approach.

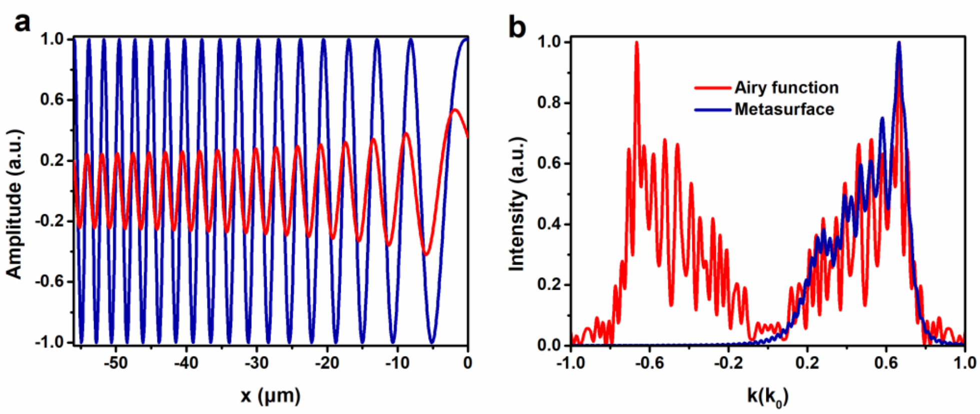

Figure S2. (a) The complex amplitude profiles of Airy function and wave function of metasurface. (b) Intensity cross profiles of the Airy function and wave function of metasurface in *k*-space.

4. Calculated Numerical Apertures and peak positions in *k*-space

The calculated numerical apertures (NAs) and peak positions in *k*-space at different wavelengths and with different radii of metasurfaces are presented in Figure S3a and S3b, respectively. The results further confirm that the NAs can conform well to the peak positions of the highest peaks in *k*-space after Fourier transformation.

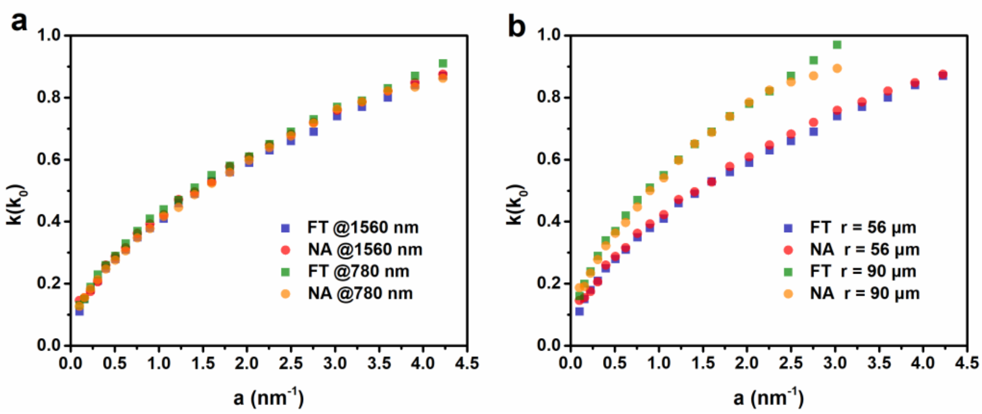

Figure S3. (a) The calculated NAs and peak positions in *k*-space at different wavelengths. (b) The calculated NAs and peak positions in *k*-space with different radii of metasurfaces.

5. Setups for characterizing the multiplexed nondiffracting nonlinear metasurfaces

A linear polarizer (LP) and a quarter-wave plate (QWP) were combined to generate the incident circularly polarized light, the metasurface was illuminated at normal incidence and the generated linear and nonlinear nondiffracting beams were filtered by another combination of a QWP and an LP, then collected with an objective, a tube lens and a scientific camera. The objective, tube lens and camera were all integrated on a XYZ translation stage to scan the intensity profiles of the nondiffracting beams with a step of 1 μm along *z*-direction as shown in Figure S4. For measurement of topological charges of SHG higher-order Bessel beams, a ±1 order phase mask was placed behind the tube lens, and a lens (with focal length of 75 mm) was used to amplify the imaging as shown in Figure S5.