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Supporting Information

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High-Performance Broadband Circularly Polarized Beam Deflector by Mirror Effect of Multinanorod Metasurfaces

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1. Theoretical formulations

The theoretical formulations for deriving generalized Snell's law (GSL) with arbitrary incidence by the mirror effect are shown as follows. We first carry out the operation when linear-polarizer operator imposes on an arbitrarily polarized incident light denoted by Jones Matrix, then obtain the complex transmittances for right circularly polarized (RCP) and left circularly polarized (LCP) anomalous lights. After Fourier transformation, the angle expression with the form of Snell's law is derived. It should be noticed that the Fourier transformation step needs the specific expression of the spatial distribution for polarizers. This analysis can be generalized to other applications such as optical lens and curved beams.

Considering linear polarizer may be imperfect, we introduce an operator P to generalize the description of linear polarizer as

$$P\left|L\right\rangle = \alpha\left|L\right\rangle + \beta\left|R\right\rangle e^{-i2\varphi}, \tag{2}$$

$$P\left|R\right\rangle = \alpha\left|R\right\rangle + \beta\left|L\right\rangle e^{i2\varphi}. \tag{3}$$

The operator P can be rewritten as the matrix expression by regarding the orthogonal basis of $\left|L\right\rangle$ and $\left|R\right\rangle$.

$$P = P_O + P_I = \begin{bmatrix} \alpha + \beta\cos 2\varphi & \beta\sin 2\varphi \\ \beta\sin 2\varphi & \alpha - \beta\cos 2\varphi \end{bmatrix}, \tag{4}$$

where the identity matrix P_O and the mirror effect P_I can be written as $P_O = \alpha\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $P_I = \beta\begin{bmatrix} \cos 2\varphi & \sin 2\varphi \\ \sin 2\varphi & -\cos 2\varphi \end{bmatrix}$, respectively. The operator can degenerate into normal Jones matrix for a perfect linear polarizer $\alpha = \beta = 0.5$:

$$P = \begin{bmatrix} \cos^2\varphi & \sin\varphi\cos\varphi \\ \sin\varphi\cos\varphi & \sin^2\varphi \end{bmatrix}. \tag{5}$$

Assuming an arbitrary polarized incident light as Jones matrix $\lvert J\rangle = \begin{bmatrix} A \\ Be^{i\delta} \end{bmatrix}$, we can achieve a succinct expression for the transmitted light by multiplying *P* before the $\lvert J\rangle$.

$$\lvert E_s\rangle = \frac{1}{2}\alpha\lvert J\rangle + \frac{1}{4}\beta(A - iBe^{i\delta})e^{i2\varphi}\lvert L\rangle + \frac{1}{4}\beta(A + iBe^{i\delta})e^{-i2\varphi}\lvert R\rangle, \tag{6}$$

where $\lvert E_s\rangle$ means the state of transmitted light. The first term represents for the unconverted object. The second and third terms represent the mirror image whose helicity and phase are symmetrically transformed as shown in **Figure 1**. To further calculate the complex transmittance for LCP light $\tilde{t}_L$ and RCP light $\tilde{t}_R$, we can do the product operation

$$\tilde{t}_L = \langle L \vert E_t\rangle = \frac{1}{4}\beta(A - iBe^{i\delta})e^{i2\varphi}, \tag{7}$$

$$\tilde{t}_R = \langle R \vert E_t\rangle = \frac{1}{4}\beta(A + iBe^{i\delta})e^{-i2\varphi}. \tag{8}$$

The real part is the transmittance or efficiency of anomalous refractive light, and the imaginary part indicates the spatial frequency of scattered light. In the linearly arranged situation in **Figure 2**, $\varphi(x)$ can be specified as $\varphi(x) = \pi\frac{x}{L}$. If a beam with spatial frequency u_λ is incident on the arrays of polarizers, the transmitted anomalous light has the expression $e^{i2\pi u_\lambda x}(\tilde{t}_R + \tilde{t}_L)$, which can be Fourier transformed as:

$$F\{e^{i2\pi u_\lambda x}(\tilde{t}_R + \tilde{t}_L)\} = a_R\delta(u - (u_\lambda - \frac{1}{L})) + a_L\delta(u - (u_\lambda + \frac{1}{L})), \tag{9}$$

where u is the spatial frequency of converted light. a_L and a_R are the real amplitudes for anomalous LCP and RCP lights. Therefore, the relation between incident and transmitted lights can be written as:

$$n_2\sin\theta_t^L = n_1\sin\theta_i + \frac{\lambda}{L}, \tag{10}$$

$$n_2\sin\theta_t^R = n_1\sin\theta_i - \frac{\lambda}{L}, \tag{11}$$

where θ_t^L and θ_t^R represent the transmitted angles for LCP and RCP lights. Similarly, we can derive the GSL for reflective situation:

$$\sin\theta_r^L = \sin\theta_i + \frac{\lambda}{nL}, \tag{12}$$

$$\sin\theta_r^R = \sin\theta_i - \frac{\lambda}{nL}, \qquad (13)$$

where θ_r^L and θ_r^R represent the reflective angles for LCP and RCP lights.

2. Anomalous reflective lights

In reflective side, the nanorod also plays the role of an imperfect polarizer. When the polarization of incident light is along with the orientation of the nanorod, the incident light would be more or less reflected. Thus, there is also a coefficiency β for nanorods in reflective side, whose maximum value also is 0.5. **Figure S1** shows the calculated amplitude transmittance of the anomalous reflective light for different multi-nanorod metasurfaces. It has similar results with that of the anomalous refracted light (**Figure 2** (b)). For the four-nanorod case, the reflective amplitude maintains high value from 200 THz to 400 THz, compared with other three metasurfaces. It also has a great blue shift effect whose central frequency nears 300 THz. Hence the anomalous light in reflective side has the similar properties, including polarization and amplitude, as the light in refractive side. The simulated electric field distribution of both reflective and refractive cases by one- and four-nanorod metasurfaces are shown in **Figure S2**. The anomalous intensity has been obviously improved for four-nanorod metasurface compared with other metasurfaces from 250 THz to 350 THz.

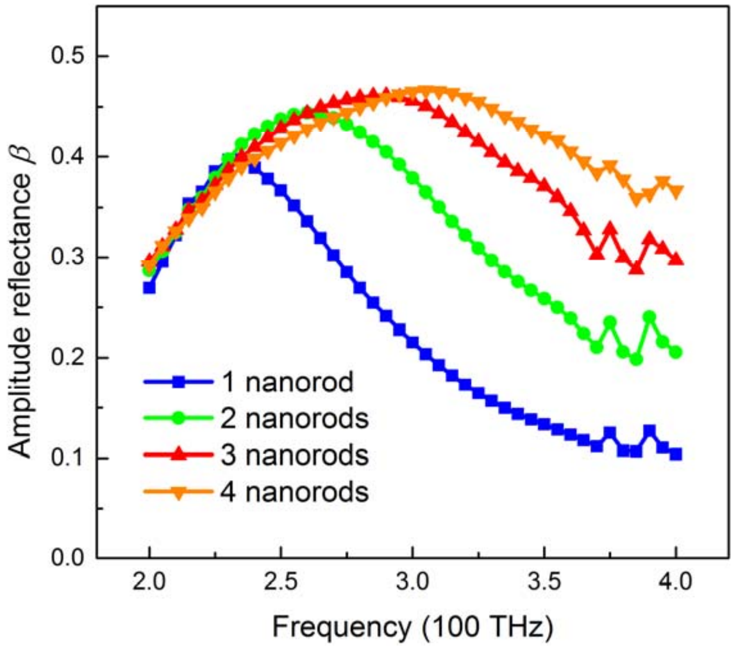

Figure S1 Calculated amplitude transmittance of the anomalous reflective light for different multi-nanorod metasurfaces. The four-nanorod metasurfaces has the broadest band with the dramatic blue shift.

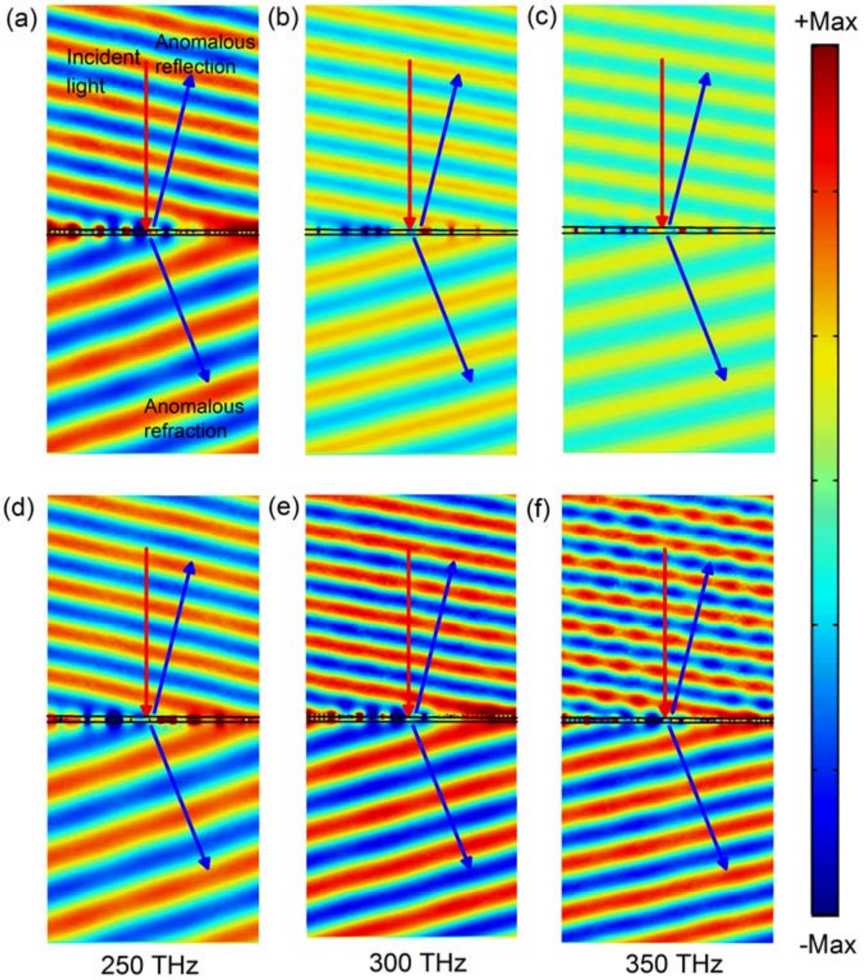

Figure S2 Electric fields of anomalous reflection and refraction by simulations in COMSOL 4.3a. (a), (b) and (c) are one-nanorod metasurfaces when incident frequencies are 250 THz 300 THz and 350 THz. (d) (e) and (f) are scattering lights by four-nanorod metasurfaces in the same frequency interval as (a) (b) and (c). All the incident lights are LCP with same amplitude.

3. Broadband polarization splitter

When incident polarization is not purely CP, the proposed metasurfaces could be regarded as a beam splitter, who can separate two opposite helicity CP lights into different directions. It can be proved that the anomalous amplitude ratio a_R / a_L for RCP and LCP lights is fixed for any certain incident polarization, even if the incident wavelength derivates the resonant wavelength of the metasurfaces. According to the cross section of Poincare sphere in **Figure 6** (d), the length of PL and PR can be expressed as:

$$PL = \sqrt{2}\sqrt{1-\sin(2\chi)}\,, \tag{14}$$

$$PR = \sqrt{2}\sqrt{1+\sin(2\chi)}\,, \tag{15}$$

where the radius of the sphere is 1. On the Poincare sphere, sin(2χ) can be specifically denoted as:

$$\sin 2\chi = \frac{2AB}{A^2+B^2}\sin\delta . \tag{16}$$

By substituting Eq. (16) into Eqs. (14) and (15), we can obtain the expression for *PR* and *PL*:

$$PL = \sqrt{2}\sqrt{1-\frac{2AB}{A^2+B^2}\sin\delta}\ , \tag{17}$$

$$PR = \sqrt{2}\sqrt{1+\frac{2AB}{A^2+B^2}\sin\delta}\ , \tag{18}$$

Comparing with the expressions for a_L and a_R , we can easily find the relationship:

$$a_R / a_L = PL / PR\ , \tag{19}$$

which does not contain the conversion efficiency β. This relation is maintained even the conversion efficiency β is low. It indicates that our proposed metasurfaces is a broadband beam splitter. The amplitude ratio of RCP and LCP anomalous lights is only dependent on the incident polarization. To further discuss the broadband effect, we caculated the value of a_R / a_L for comparison with the simulated results as shown in **Figure S3**. The orange line represents for the ratio of the *PL*/*PR* on Poincare sphere (*A*=*B*). The markers with different shapes are the simulated amplitudes for different polarizations from 200 THz to 400 THz by changing δ. Results show that the amplitude ratios are in good agreement with the simulated results.

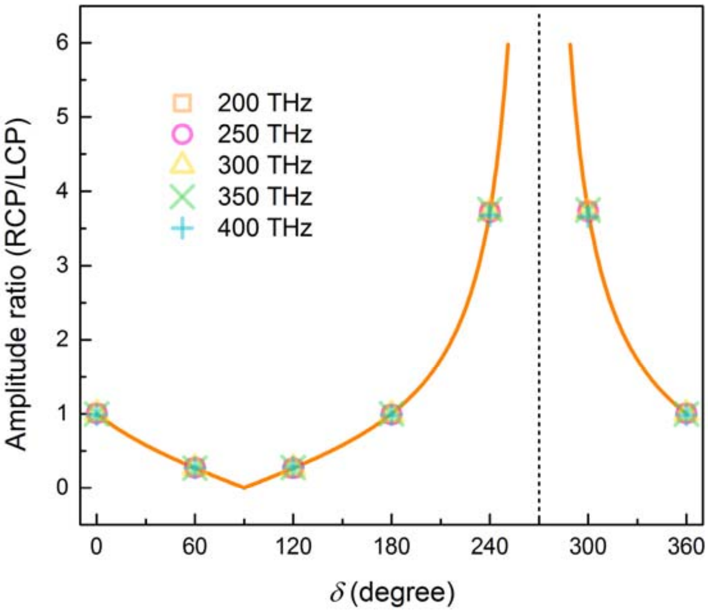

Figure S3 Calculated the value of a_R / a_L and the simulated results.