Supporting Information

Optical Information Multiplexing with Nonlinear Coding Metasurfaces

Menglin Ma[1], Zhi Li[1], Wenwei Liu[1], Chengchun Tang[2], Zhancheng Li[1], Hua Cheng[1], Junjie Li[2], Shuqi Chen[1,3,4,], and Jianguo Tian[1,4]*

[1] The Key Laboratory of Weak Light Nonlinear Photonics, Ministry of Education, School of Physics and TEDA Institute of Applied Physics, Nankai University, Tianjin 300071, China.

[2] Beijing National Laboratory for Condensed Matter Physics, Institute of Physics, Chinese Academy of Sciences, P.O.Box 603, Beijing 100190, China

[3] The collaborative Innovation Center of Extreme Optics, Shanxi University, Taiyuan, Shanxi 030006, China.

[4] Renewable Energy Conversion and Storage Center, Nankai University, Tianjin 300071, China

*Corresponding Author, email: schen@nankai.edu.cn

1. Digital coding for various symmetric meta-atoms.

Using SRRs with one-fold rotational symmetry as the meta-atoms, we proposed the methodology of applying the convenient digital signal processing to design the multidimensional information storage channels in nonlinear optical processes. Actually, this design methodology is general and not confined to the SRRs with one-fold rotational symmetry. In Table S1, we show the digital coding of meta-atoms with various rotational symmetries under *n*th harmonic generation. It is visible from the table that in the range from the first to fifth harmonic generation, one-fold rotational symmetric structure can implement 1-bit, 2-bit and 3-bit coding, two- and three-fold rotational symmetric structures can implement 1-bit and 2-bit coding, and four-fold rotational symmetric structure can implement 1-bit coding. Consequently, we choose the SRRs as meta-atoms, because only the structures with one-fold rotational symmetry can realize three kinds of digital coding in the range of FF to THG. With the enhancement of nonlinear conversion efficiency by utilizing the systems consist of high nonlinear mateial,[1-2] higher harmonics could be promising to be introduced as new information storage channels by the way of digital coding. Therefore, by choosing the appropriate combination of digital coding and harmonic orders, the structures with various rotational symmetries could be utilized to design the nonlinear coding metasurfaces for the realization of optical information multiplexing.

Table S1. Digital coding for various symmetric meta-atoms

Harmonic generation	spin state σ	C1[a)] 	C2[a)] 	C3[a)] 	C4[a)] 
1st	+1	—	—	—	—
	-1	2-bit	2-bit	—	—
2nd	+1	3-bit	—	—	—
	-1		—	2-bit	—
3rd	+1	2-bit	2-bit	—	—
	-1	1-bit	1-bit	—	1-bit
4th	+1		—	2-bit	—
	-1		—	—	—
5th	+1	1-bit	1-bit	—	1-bit
	-1			1-bit	—

[a)]C1–C4: one- to four-fold rotational symmetry of the meta-atoms.

2. Theoretical analysis for the focusing performances of multifocal metalens.

In order to determine the effect of paraxial condition on performances of the multifocal metalens, we performed quantitative analysis based on the focusing theory. First, in order to achieve perfect focusing for THG, the continuous phase distribution ϕ_{THG} needs to satisfy $\phi_{THG}\left(r\right)=-k_{THG}\left(\sqrt{r^2+f_{THG}{}^2}-f_{THG}\right)$, then the focusing situations of FF and SHG should be strictly deduced from Eq. (1) (i.e. $\phi_{FF}\left(r\right)=-k_{FF}\left(\sqrt{r^2+f_{FF}{}^2}-f_{FF}\right)$, $\phi_{SHG}\left(r\right)=-k_{SHG}\left(\sqrt{r^2+f_{SHG}{}^2}-f_{SHG}\right)$). Substituting the relations $\phi_{THG}(r)=2\phi_{FF}(r)=4\phi_{SHG}(r)=4\varphi(r)$,[3] and $k_{THG}=3k_{FF}=3k_{SHG}/2$ into the above formulas, we can obtain:

$$f_{FF}=\frac{4r^2-\left[3\left(\sqrt{r^2+f_{THG}{}^2}-f_{THG}\right)\right]^2}{12\left(\sqrt{r^2+f_{THG}{}^2}-f_{THG}\right)}, \quad \text{(S1)}$$

$$f_{SHG}=\frac{64r^2-\left[3\left(\sqrt{r^2+f_{THG}{}^2}-f_{THG}\right)\right]^2}{48\left(\sqrt{r^2+f_{THG}{}^2}-f_{THG}\right)}, \quad \text{(S2)}$$

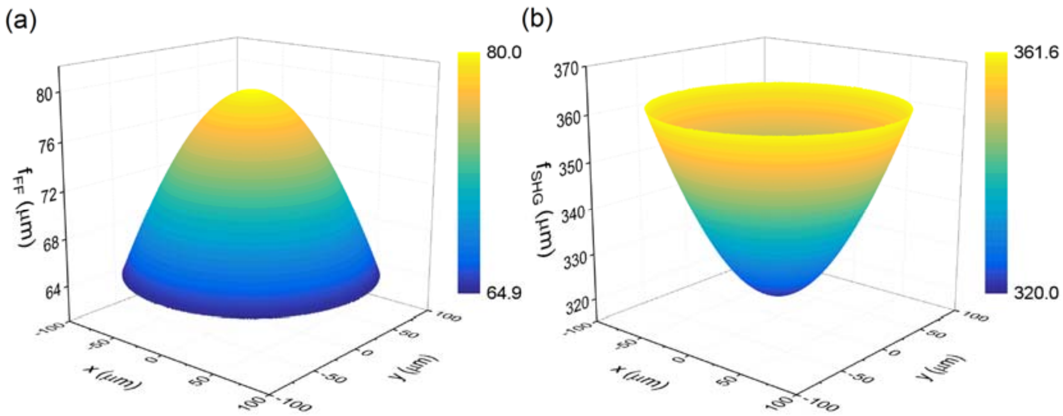

Figure S1. The distributions of focal lengths for FF and SHG under the condition that perfect focusing on the THG channel is achieved. (a) The distribution of focal length for the FF. As the radius r increases from 0 to 100 µm, the f_{FF} decreases from 80 µm to 64.9 µm. (b) The distribution of focal length for the SHG. As the radius r increases from 0 to 100 µm, the f_{SHG} increases from 320 µm to 361.6 µm.

In the case of f_{THG} = 120µm, the profiles of f_{FF} and f_{SHG} are shown in Figure S1. Evidently, the metalens, which can achieve perfect focusing under the THG channel, cannot obtain ideal spot-focusing for the channels of FF and SHG. When the radius r increases from 0 to 100 µm, the f_{FF} decreases from 80 µm to 64.9 µm, whereas the f_{SHG} increases from 320 µm to 361.6 µm. Therefore, the sample with smaller radius will be expected to achieve more excellent focusing performances for the FF and SHG.

3. The multi-channel information storage by nonlinear coding metasurface.

The extra degree of freedom among the three channels in nonlinear coding metasurfaces reduces the correlations among the three information storage channels, which is beneficial for the realization of multi-channel optical information storage. In order to verify the feasibility of multi-channel

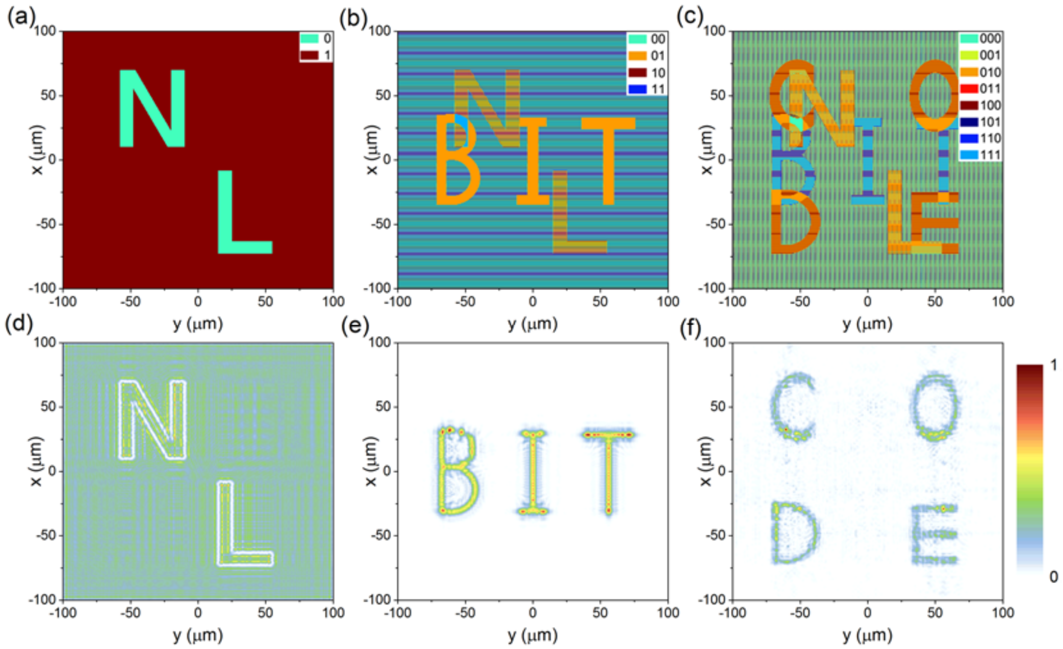

Figure S2. The multi-channel information storage by nonlinear coding metasurface. (a-c) The coding patterns of the metasurface for the (a) THG, (b) FF and (c) SHG in the *x-y* plane. (d-f) Numerically calculated real space images under the channels of (d) THG, (e) FF and (f) SHG in the *x-y* plane.

information storage in nonlinear coding metasurfaces, we theoretically design the nonlinear coding metasurface which stores different information in different channels, and ensures the information remains independent of each other. Figure S2 shows the coding patterns and numerically calculated intensity profiles of the nonlinear coding metasurface under three different channels, in which we stored the abbreviation ‘NL’ of nonlinear in the channel of THG, the word ‘BIT’ in the FF channel, and the word ‘CODE’ in the THG channel, respectively. From the numerically calculated results of this nonlinear coding metasurface, the intensity profiles in the far field are highly consistent with the multiplexed functionality we theoretically designed, which confirms it is viable to implement multi-channel optical information storage by exploiting nonlinear coding metasurfaces.

References

[1] H. Liu, C. Guo, G. Vampa, J. L. Zhang, T. Sarmiento, M. Xiao, P. H. Bucksbaum, J. Vučković, S. Fan, D. A. Reis, *Nat. Phys.* **2018**, *14*, 1006.

[2] T. Shibanuma, G. Grinblat, P. Albella, S. A. Maier, *Nano Lett.* **2017**, *17*, 2647.

[3] G. Li, S. Chen, N. Pholchai, B. Reineke, P. W. Wong, E. Y. Pun, K. W. Cheah, T. Zentgraf, S. Zhang, *Nat. Mater.* **2015**, *14*, 607.